## Lab 3: Imaging Asteroids

## Finding Asteroids

There are now more than 700,000 minor planets (asteroids, comets, and Kuiper Belt Objects) cataloged by the Minor Planet Center, and more are being added daily. When an asteroid is first discovered, it is given a preliminary name consisting of the year of discovery, followed by a letter indicating the half-month of discovery (e.g. A for Jan 115, B for Jan 16-31, etc., but skipping the letter I). When more than one asteroid is discovered in a given half-month, it is given a second letter in sequence (i.e. 2006 CA, followed by 2006 CB ). When all letters through Z are exhausted, numbers are added (i.e. 2006 CZ is followed by 2006 CA1, followed by 2006 CB1, etc.). However, some letters and numbers are skipped, for reasons I do not understand. Possibly this is because some "discoveries" were given a designation and then later found to be space junk, or for some other reason not a real asteroid. The total number of asteroids discovered in the last half of January, 2017 (i.e. with designations starting with 2017 B) was 1360 ! Most of these are very faint (greater than magnitude 17). Once an asteroid has been observed many times over more than one orbit, it is given a permanent designation which is a sequential number followed by a name. The name is one given by the discoverer, but must be agreed upon by the International Astronomy Union. The first asteroid is 1 Ceres, the second is 2 Pallas, etc. As of this time, there are more than 480,000 numbered asteroids, but only about 20,000 have names. Some of the names are strange or whimsical. Use the steps listed below to look up the names of the following asteroids and write them into your lab book: (2309, 2404, 3552, 3568, 4231).

We are going to start with some bright and easy to find asteroids. As a class we will observe one asteroid from among the first 100 numbered asteroids, and then you will each choose another from among the fainter ones (numbered above 1000-the higher the better). Cartes du Ciel has a list of asteroids numbered below 5000-although it also has provision for downloaded the full list from the Minor Planet Center (MPC). The 5000asteroid list has already been loaded and positions updated for February 2017. To look up a particular asteroid,

- Click on the "find" icon (looks like binoculars).
- Select the Asteroid radio button
- Type in the asteroid number and click "Filter->" for the official name
- Click "Find" to center the asteroid in the display. Note that the asteroid may be below the horizon.
- Once you have one you like, you can re-find it by just typing the name in the search box next to the "find" icon.
Cartes du Ciel does not necessarily plot asteroids. If they are not displayed (as a tiny yellow dot with its name next to it), click the "Show Asteroids" icon (three dots).

Once you have the asteroids plotted in Cartes du Ciel, click on a few that you want to observe and check their magnitudes. For the asteroids numbered 1-100, try to find one
greater than magnitude 12. For the asteroids numbered greater than 1000 , try to find one greater than magnitude 15. Anything fainter than 15 may be too faint for Newark skies.

## Taking the Data

These instructions assume that you have successfully connected to the telescope and camera, the camera is at a stable temperature lower than -20 C , and that the telescope is focused well using a 4-5 magnitude star. Once you have selected your asteroid, write down in your lab book its designation (number and name), and its listed visual or red magnitude. You may find its J2000 coordinates useful to write down, also. Point the telescope at its location and take an image in the Luminance filter to verify that you have the correct location. Identify the stars in the field, and use Astrometry.net to verify that the telescope coordinates are the expected ones. Write down the image center coordinates, focal length and orientation (position angle) that Astrometry.net gives, for use later in another program we will use: Astrometrica.

Once you are satisfied that you are correctly pointed (you will probably be able to identify the asteroid, since these are quite bright), set up the image sequence to take a series of 3 Luminance-filter images of 20 s each, using the name of the asteroid, followed by a "_1" as your file name. For example, if your asteroid is Arago, use Arago_1 as the name. Come back to the asteroid at least two more times in the next two hours or so, and take two more sets of images with the name Arago_2 and Arago_3. The image sets should be separated by about $20-45 \mathrm{~min}$. Others will be taking their images in the gaps between your images, so a certain amount of efficiency is required.

When you are done with the data taking, you should have three image sets on your asteroid numbered greater than 1000 . We also want to calibrate these images. As a class, we will take the necessary bias and dark frames, with the names Bias and Dark, as we did in lab 2. We will also take our first flat frames.

## Flat Frames

Whenever you wish to take nice deep-sky photographs, or when you want to do photometry (using the CCD data to determine accurate relative brightnesses), you will have to take a series of flat frames in each filter you are using for your data. In this case, we will use only the Luminance filter, so we will take Luminance flat frames. The basic idea is to measure the vignetting (brightness variations in the image due to partial blocking of the main mirror of the telescope), or correct for "dust donuts," by pointing at an area of uniform brightness and taking images at an exposure that will give about $20,000-40,000$ counts in each pixel of the image. You will be shown how to take a sequence of 10-20 such images, with the name of Flat, to make a nice, uniform illumination of the CCD. Any non-uniformities that we see in the image are due to the vignetting in the telescope, or dust on the optics. As a class, we will take our images at dusk before class or at the end of the night before closing up the telescope (there are several options which will be discussed in class). Unlike the Bias and Dark frames, which are subtracted from the data, the data are divided by the (normalized) Flat frame. For example, say the brightest part of the Flat frame is at brightness 30,000, and the darkest part is at 27,000 . Then the Flat frame is first normalized (divided by 30,000 ) to
give 1 for the brightest part and $0.9(27,000 / 30,000)$ for the darkest part. Then the data are divided by the normalized flat, so that the brightest part of the data frame is divided by 1 (i.e. not changed) while the darkest part of the data frame is divided by 0.9 (i.e. boosted in brightness by $10 \%$ ). As the name implies, this flattens the data frame, making its sensitivity uniform.

## Analyzing the Data

As a class, we will first do the calibration and create master frames. To do this, we open the Process/Set Calibration... menu item in MaxIm DL, and select the directory where we stored our Bias, Dark and Flat frames. The "Auto-Generate" button should then give us the correct list of frames of each type. Once we are satisfied, we will click the "Replace with Masters" button to perform the analysis and create single master frames of each type (one each for Bias, Dark, and Flat). Read in the Flat master frame and take a look at it. You should see an overall pattern of a bright doughnut-like shape, with darker corners. When applied as described above, this brightness pattern will be used to correct for brightness variations in the data images. By using the "Replace with Masters" button, we converted our dozens of Bias, Dark, and Flat frames into three calibration frames that we will then upload to the Data Depository so that you can use them to calibrate your data.

To process your data, you will first set the calibration to use the three master frames just created. Then use Batch Save and Convert from the File menu in MaxIm DL to apply the calibration, and write your calibrated frames to another directory. You will use these calibrated frames for all further analysis. The first step is to combine the images for each of the three sequences on each asteroid. We combine images to improve the signal to noise, and to allow us to throw out images that might have been affected by poorer than normal telescope tracking. After combining, you should have three calibrated and combined images for each asteroid, taken at different times over 1 or 2 hours. The stars will all be fixed, but the asteroid will have moved. After combining, using the example above, you would have three images, Arago_1, Arago_2, and Arago_3. Now read in all three of these and choose Align from the process menu. Follow the procedure to align these images, using the middle one as the reference. You can then view them as an animation if you wish (choose Animate... from the View menu) just to check that the alignment went well and that you can see the asteroid move. If your asteroid is faint, you may not be able to find it, but Astrometrica can. SAVE your three aligned images for each asteroid, using Save As... from the File menu, and be sure to select Size Format as 16-bit Int. It will probably be set to IEEE float, which is NOT a format that Astrometrica can read.

## Using Astrometrica for Astrometry and Photometry

The last step is to read your images into Astrometrica for measuring the asteroid positions and brightness. First, open Astrometrica and click the choice to delete the existing log file. Then read all three images for a given asteroid - the times and dates should all be okay, so you can click OK to each. Now select Moving Object Detection from the Astrometry menu. The program will ask you for coordinates, so check that they are what they should be (the coordinates that you wrote down from Pinpoint when you were taking
data). If they are not correct, either use the coordinates you wrote down from Cartes du Ciel, or if you did not record them, go back to Cartes du Ciel, set it for the correct date and time, and check the coordinates of your asteroid there.

Once Astrometrica has the coordinates, it will do the usual attempt to solve for the star positions. If it cannot, use manual mode and type the orientation angle from Astrometry.net, but with the opposite sign. Also check the focal length. After the usual effort, Astrometrica should get the solution and will then automatically detect moving objects. It will probably identify $10-100$ possible objects! If your asteroid is bright enough, it will find it as the first moving object. We will discuss how to use the moving object feature to designate and accept moving objects. Once accepted, the information on the asteroid will be placed in a minor planet center (MPC) report. You can list that by selecting "View MPC report file" from the File menu. This report file can be sent directly to the minor planet center to be used in their database! This information is actually of scientific interest. In addition to the position, the brightness of the asteroid will also be listed. Compare the magnitude you found with the predicted magnitude that you wrote down from Cartes du Ciel.

Finally, choose View Log File from the File menu and write down the speed and position angle of the motion of the asteroid. Use the "Track and Stack" feature of Astrometrica to combine your three images into a single image that follows the asteroid and allows the stars to trail. Save this image and include it in your report. On the image, add text that indicates the asteroid name and number, the center (middle image) coordinates, the magnitude, the speed (in arcsec/minute), and direction of motion.

## Results

## Asteroid Distance and Orbital Radius

The motion of asteroids is complicated, because both the Earth and the asteroid are moving in their orbits. Therefore, any observation over a short period such as a couple of hours cannot give an accurate idea of the asteroid's distance. The asteroid may be near a stationary point, or even be in retrograde motion. However, most of the lower numbered asteroids such as the ones you have imaged are in circular orbits in the asteroid belt. In that case, we can use the speed of motion and the brightness to tell us something about the asteroid's orbital distance and size. We will do this exercise for the brighter asteroid, but if you wish you can repeat it for the fainter one as well. The geometry of this situation is shown in Figure 2.


Figure 2: Geometry of Sun, Earth, and Asteroid, where the orbits are considered to be circular, and the distances are in astronomical units (AU). The distance from Sun to Earth is 1 AU , the distance from Earth to the asteroid is d, and the distance from Sun to asteroid is r .

The angle (as seen from Earth) between the Sun and the asteroid is SA, which we can measure. The line of sight to the asteroid is along $d$, so the plane of the sky is perpendicular to this direction, as shown. The orbital velocities of Earth and asteroid are $v_{e}$ and $v_{a}$, respectively, and the components of these velocities in the direction of the plane of the sky are $v_{e s}$ and $v_{a s}$. The asteroid appears to move in the sky with speed $v=$ $\left(v_{a s}-v_{e s}\right)$. Now the speed of a circular orbit is just the distance around the orbit ( $2 \pi r$ ) divided by the period, $P$. Let's use speed units of AU/yr, to save us some work. Then the orbital speed is just $v=2 \pi r / P$, so $v_{e}=2 \pi$ (since $r$ is 1 AU and $P=1 \mathrm{yr}$ ) and $v_{a}=2 \pi r / P$. where $r$ is the distance from Sun to asteroid, and $P$ is the orbital period of the asteroid in years. But we know how periods depend on distance from the Sun for circular orbits (from Kepler's Law of Periods), $P^{2}=r^{3}$. So $v_{a}=2 \pi r / r^{3 / 2}=2 \pi r^{-1 / 2}$.

Now what about the angles in the situation? Because $v_{a}$ is perpendicular to $r$, and $v_{a s}$ is perpendicular to $d$ (by definition), the angle between $v_{a}$ and $v_{a s}$ is the same as the SunEarth angle SE. Thus, $v_{a s}=v_{a} \cos$ SE. Likewise, because $v_{e}$ is perpendicular to the SunEarth line, and $v_{e s}$ is perpendicular to $d$ (by definition), the angle between $v_{e}$ and $v_{e s}$ is 180 -SA , so $v_{e s}=v_{e} \cos (180-\mathrm{SA})=-v_{e} \cos \mathrm{SA}$.

The final velocity we want, then, is $v=\left(v_{a s}-v_{e s}\right)=\left(v_{a} \cos \mathrm{SE}+v_{e} \cos \mathrm{SA}\right)$. Unfortunately, we do not know the angle SE. However, using the Law of Sines, we can replace it with an expression involving the angle we do know, SA, since

$$
r / \sin \mathrm{SA}=1 / \sin \mathrm{SE}
$$

so $\cos \mathrm{SE}=\left(1-\sin ^{2} \mathrm{SE}\right)^{1 / 2}=\left(1-\sin ^{2} \mathrm{SA} / \mathrm{r}^{2}\right)^{1 / 2}$. Substitute that into the above expression for v , and use the fact that $v_{e}=2 \pi$ and $v_{a}=2 \pi r^{-1 / 2}$, to get:

$$
v=2 \pi\left[\left(1 / r-\sin ^{2} \mathrm{SA} / \mathrm{r}^{3}\right)^{1 / 2}+\cos \mathrm{SA}\right]
$$

This is the difference in velocities in the plane of the sky between Earth and the asteroid, in terms of the distance of the asteroid from the Sun, and the angle between the Sun and asteroid as seen from Earth. What we measure, however, is not the velocity but only the angular velocity, call it $v_{\text {ang }}$, in arcsec/minute. Now, given $v$ in $\mathrm{AU} / \mathrm{yr}$, we can find $v_{\text {ang }}$ in $(\operatorname{arcsec} / \mathrm{yr})$ by $v_{\mathrm{ang}}(" / \mathrm{yr})=[v(\mathrm{AU} / \mathrm{yr}) / d(\mathrm{AU})] * 206265 " /$ radian. There are $365.25 * 24^{*} 60$ $=513,000$ minutes in a year, so

$$
\begin{equation*}
v_{\text {ang }}(" / \mathrm{min})=[v(\mathrm{AU} / \mathrm{yr}) / d(\mathrm{AU})]^{* 206265 / 513000}=0.4 v / d . \tag{2}
\end{equation*}
$$

Inserting $v$ into this expression and solving for $d$, we end up with the remarkably simple relation for the distance to the asteroid:

$$
\begin{equation*}
d=\left[0.8 \pi / v_{\text {ang }}(" / \mathrm{min})\right]\left[\left(1 / r-\sin ^{2} \mathrm{SA} / r^{3}\right)^{1 / 2}+\cos \mathrm{SA}\right] . \tag{3}
\end{equation*}
$$

The last thing to do is use the Law of Cosines to relate $d$ and $r$ in terms of SA:

$$
\begin{equation*}
r=\left[1+d^{2}-2 d \cos \mathrm{SA}\right]^{1 / 2} \tag{4}
\end{equation*}
$$

Inserting (3) into (4) gives the final expression for orbital distance to the asteroid in terms of measurable quantities SA (angle between Sun and asteroid) and $v_{\text {ang }}$ (angular speed of the asteroid in the plane of the sky, in arcsec/minute). The problem is, doing that leaves us with a nasty function of $r$ that we do not know how to solve. However, we can simply try a range of distances $r$ on the right side of (4) and the one that equals $r$ on the left is the solution. In other words, plot the function on the right side, and the function $r=r$, and where they cross is the solution. I have written a Python program that does that, called AST_DIST. You simply give it your measured values of $v_{\text {ang }}$ and SA, and it prints the distance to the asteroid and the orbital radius of the asteroid.

Run AST_DIST, using your values of $v_{\text {ang }}$ and SA for the asteroid numbered less than 100 , and write down the returned values for $r$ and $d$. Compare them with the corresponding values given by Cartes du Ciel. They should agree fairly closely, depending on how non-circular (eccentric) the asteroid orbit is. Write down your values of $r$ and $d$. You will use them in the next section.

## Asteroid Albedo and Size

Once we know the approximate distance to our asteroid, we can use the measured magnitude to determine the actual size of the asteroid. You may think that, for a given distance, a bright asteroid would be a big one, and a dim one a small one. That is true if they all reflect the same fraction of light (called the albedo). However, they are not all the same. Some are reflective, like ice or snow, while some are black as coal. The following description is from "Astronomy \& Astrophysics, $4^{\text {th }}$ Edition" by Zeilik and Gregory ${ }^{1}$ :

The...solar [power] falling on an asteroid is

$$
\left(L_{\mathrm{sun}} / 4 \pi D^{2}\right) \pi R^{2}
$$

where $L_{\text {sun }}$ is the luminosity of the Sun, $D$ is the distance from the $\operatorname{Sun}$, and $R$ is the radius of the asteroid. A fraction $A$ (the albedo) is reflected back into space.

[^0]If the Earth is a distance $d$ from the asteroid, the flux of reflected light at the Earth is

$$
\begin{equation*}
F_{\mathrm{vis}}=\left(L_{\mathrm{sun}} / 4 \pi D^{2}\right) \pi R^{2}\left(A / 4 \pi d^{2}\right) \tag{4}
\end{equation*}
$$

We can measure this flux and so determine the quantity $R^{2} A$. The fraction of energy absorbed, $1-A$, heats up the asteroid and is re-emitted into space as infrared radiation. We can observe this infrared flux at the Earth. The ratio of visible to infrared flux is

$$
F_{\mathrm{vis}} / F_{\mathrm{IR}}=A(1-A)
$$

From this measurement, we can determine $A$, and then use this value with the previous determination of $R^{2} A$ to calculate $R$.

You can look up the albedo for many named asteroids at the Wikipedia web pages. High numbers mean a reflective surface, while low numbers mean a dark surface. Look up the albedo for our asteroid and write it down in your log book.

You measured the flux of your asteroid by measuring the visual magnitude in Astrometrica, but how does flux relate to magnitude. Recall that magnitudes only work as magnitude differences, which relate to flux ratios. So a single magnitude does us no good. We need to compare it with something. Since it is the Sun that is providing the light by which we see the asteroid, let's use the Sun as a comparison. The basic equation is:

$$
\begin{equation*}
m_{1}-m_{2}=2.5 \log F_{2} / F_{1} \tag{5}
\end{equation*}
$$

where $m_{1}, m_{2}$ are two magnitudes, and $F_{1}, F_{2}$ are the two corresponding fluxes. Flux is the luminosity divided by the area of a sphere centered at the star (or body) and of a radius equal to the distance to the object, $L / 4 \pi d^{2}$. We will use $m_{1}=m$ as the apparent magnitude of the asteroid, and $m_{2}=-26.72$ as the apparent magnitude of the Sun. The flux of the Sun at Earth is $F_{2}=L_{\text {sun }} / 4 \pi r_{\text {Earth }}{ }^{2}$, where $r_{\text {Earth }}=1 \mathrm{AU}$. The flux of the asteroid, $F_{1}$, is just the value given in equation (4).

Substituting all of this into equation (5), we have

$$
m-(-26.72)=2.5 \log \left[\left(r^{2} / r^{2} \text { Earth }\right)\left(4 d^{2} / R^{2} A\right)\right]
$$

All you need to do, then, is insert your measured value of $m$, your calculated values of $d$ and $r$ (in AU), the value $r_{\text {Earth }}=1$ (in AU), and the albedo, $A$, that you got from the albedo table. Solve this equation for the radius of the asteroid, $R$. Compare your calculated value with the true size of your asteroid, if you can find it on Wikipedia. (Try entering the number and name of your asteroid in the search, e.g. " 24 Themis".)

## Conclusion

You have made some rather simple measurements of asteroids-their position, their speed and direction of motion, their magnitude. Using these few facts, plus the asteroid's albedo, together with some geometrical relations and Kepler's Law, you were able to estimate the distance to the asteroid, its distance from the Sun, and even a measure of its size. This is the remarkable thing about the universe-it follows mathematical relationships and other physical laws that allow us to learn a lot about it from just a few careful measurements-things that we could never have imagined that we could know from just a few feeble photons.

## Bulletized Synopsis

## Purpose: Measure the position of main-belt asteroids and estimate distance and size

- Take three sets of images separated by 30-45 minutes over 1-2 hours, of two asteroids (one numbered less than 100, and one numbered greater than 1000).
- Combine the images within each set, then align with the other sets to create three aligned images for each asteroid.
- Use the images to measure the position, motion and brightness (magnitude) of the asteroid in Astrometrica.
- Use your measurements to estimate the distance and size of the brighter asteroid.


[^0]:    ${ }^{1}$ Copyright (1998), Saunders College Publishing, Harcourt Brace, New York, p133.

